Hand written letter of Paul Lorenzen to Evert Willem Beth, dated 17 August 1959
Translation and notes by Erik C. W. Krabbe

P. Lorenzen Kiel, 17/8 59
Clausewitz str. 14

Highly esteemed, dear Mr. Beth,

Maybe this letter will reach you before you will have departed for Warsaw.[1] I'm much looking forward to meet you there: for I just got your new voluminous book[2] and am now reading in it with the greatest interest.

It is so encompassing that I have of course read just a part of it – but enough to now compliment you, without any reservations, on the mathematical elegance displayed in your proofs of all important theorems – and also on the new light thrown upon philosophical and historical connections.

Your new device, the semantic tableaux, is now very nicely and clearly expounded. There is also another reason for me to be particularly interested in these tableaux – and I would be very pleased if we could discuss this in detail sometime in Warsaw. When trying to define the term "definite," which is used in my "*Einführung in die operative Logik and Mathematik* [Introduction to Operative Logic and Mathematics]," it occurred to me to investigate more closely how logical particles are used when they appear in a dialogue (between a proponent P and an opponent O). If one defines the way to make use of the logical particles in an obvious way, and if one then writes out the dialogues, then – with inessential transpositions – exactly your tableaux make their appearance.

May I illustrate this just briefly, using your example "festino"? Let the proponent P assert the logical implication $(x)[P(x)\to\neg M(x)]\wedge(Ey)[S(y)\wedge M(y)]\to(Ez)[S(z)\wedge\neg P(z)]$,[3] i.e. he is obligated to assert the conclusion when his opponent asserts the premises.

opponent	proponent
(1) $(x)[P(x)\to\neg M(x)]$	
(2) $(Ey)[S(y)\wedge M(y)]$	$(Ez)[S(z)\wedge\neg P(z)]$

For any assertion, one may always be asked to provide a "proof." If O demands a proof for the assertion P(2), P may, however, first demand a proof for O(1), O(2). A "proof" for O(2) requires the specification of an element a

[1] Lorenzen expects Beth to attend the Symposium on Foundations of Mathematics, Warsaw, 2-9 September 1959.

[2] E. W. Beth, *Foundations of Mathematics,* 1959.

[3] In formulas the "overlining" that symbolizes negation has been replaced by the use of "¬".

(3) $S(a) \wedge M(a)$	?

A “proof” for a conjunction requires that both conjuncts be asserted

(4) $S(a)$	
(5) $M(a)$	

Since also O(1) has been asserted, P may *select* any element, for instance a, so that O will then have to specify his assertion, which starts with (x), with respect to a

(5)	? (a)
(6) $P(a) \rightarrow \neg M(a)$	

Now P “proves” his assertion P(2) by

(6)	$S(a) \wedge \neg P(a)$
(7) ?	$S(a)$
(8)	$\neg P(a)$

O can now not go on casting doubt on P(7), i.e. S(a), because he has asserted it himself before. When O wants to cast doubt on P(8), then he should, since that is a negation, assert himself P(a)

(9) $P(a)$	

But then P may also assert P(a)

(9)	$P(a)$

and O will now, because of (6), have to assert $\neg M(a)$ as well

(10) $\neg M(a)$	

P may cast doubt on this assertion by asserting M(a) himself

(10)	$M(a)$

on which O is not allowed to cast doubt, having at (5) already asserted it himself. Thus assertion O(9) has been refuted – and P and his assertion have “won.”

Now one may define a formula to be “logically valid,” if there exists for it a winning strategy in this dialogue game (for elementary statements it is agreed that O may assert every elementary statement and P only those that were asserted by O before).

The existence of a winning strategy is equivalent with the existence of a closed tableau – and thus with deducibility in an appropriate logical calculus (indeed here in the intuitionistic calculus, to get the classical calculus one should somewhat modify the rules of dialogue, for instance, so that one may always add $A \vee \neg A$).

By non-finitary means, one will – I suspect – in the wake of your completeness proof be able to prove that for each formula there exists either a winning strategy for P or one for O (i.e. a counterexample model).

Consequently, it seems to me that the tableaus might be helpful to establish a good connection between the “semantic” and the “operative” view –which matter we may perhaps discuss in Warsaw.

With kindest regards – and again my congratulations on the completion of your book –

always faithfully yours

P. Lorenzen